

Effects of electrostatic fields and Casimir force on cantilever vibrations

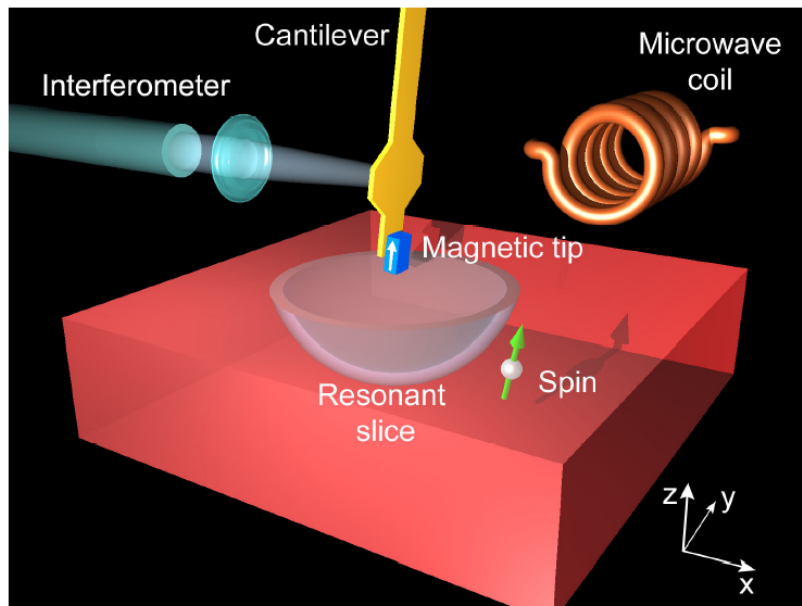
A.A. Chumak P.W. Milonni, and G. P. Berman (LANL)

The effect of an external bias voltage and fluctuating electromagnetic fields on both the fundamental frequency and damping of cantilever vibrations is considered. An external voltage induces surface charges causing cantilever-sample electrostatic attraction. A similar effect arises from charged defects in dielectrics that cause spatial fluctuations of electrostatic fields. The cantilever motion results in charge displacements giving rise to Joule losses and damping. It is shown that the dissipation increases with decreasing conductivity and thickness of the substrate, a result that is potentially useful for sample diagnostics. Fluctuating electromagnetic fields between the two surfaces also induce attractive (Casimir) forces. It is shown that the shift in the cantilever fundamental frequency due to the Casimir force is close to the shift observed in recent experiments of Stipe *et al.* Both the electrostatic and Casimir forces have a strong effect on the cantilever eigenfrequencies, and both effects depend on the geometry of the cantilever tip. We consider cylindrical, spherical, and ellipsoidal tips moving parallel to a flat sample surface. The dependence of the cantilever effective mass and vibrational frequencies on the geometry of the tip is studied both numerically and analytically.

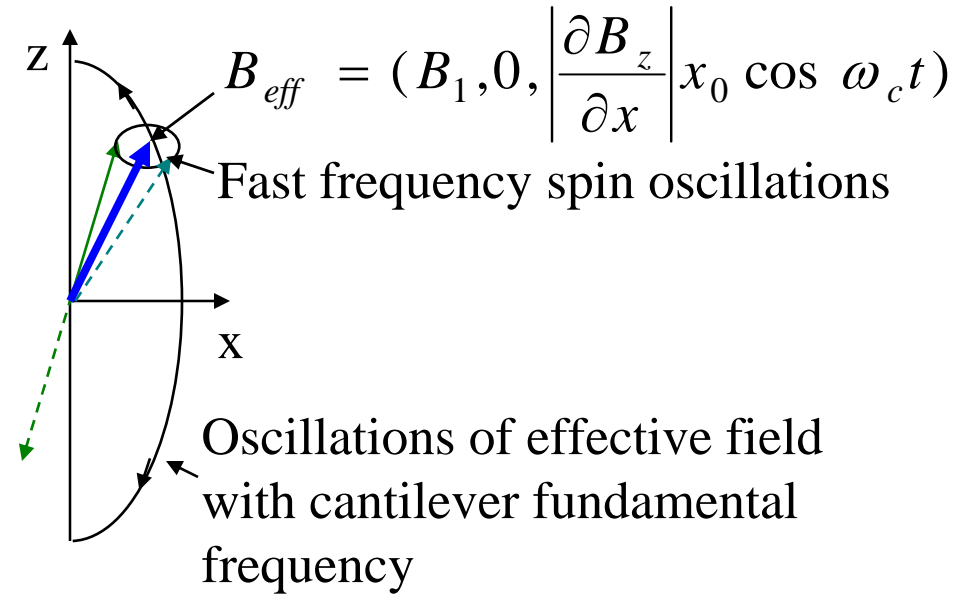
Quantum effects in a single-spin OSCAR MRFM

- **Problem:** quantum jumps can prevent measurement of spin state.

Rugar *et. al.* experiment on single spin OSCAR MRFM, IBM, NATURE, 2004



Effective magnetic field which acts on a spin



- **Spin oscillates with a cantilever frequency.**
- **Back reaction of spin on cantilever results in a cantilever frequency shift which is measured.**
- **Quantum jump appears due to magnetic noise caused by the cantilever vibrations near Rabi frequency: $\Omega_R = \gamma B_1 \gg \omega_c$.**



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Noncontact friction

(Dependences on tip-sample spacing, temperature, and bias voltage)

B.C. Stipe *et al.*, PRL **87**, 096801 (2001)

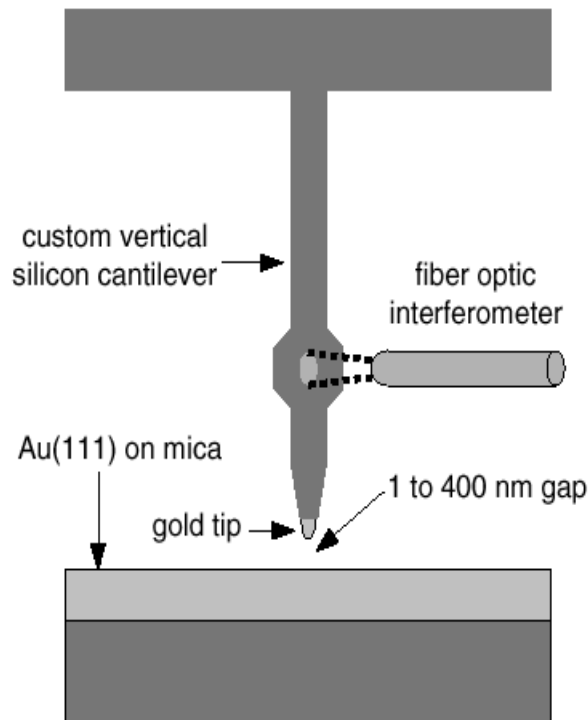


FIG. 1. Schematic diagram of the experimental setup. The cantilever's motion was detected with a fiber optic interferometer while the tip-sample spacing was controlled by piezoelectric actuators. The entire cantilever was metallized so that the tip-sample bias voltage could be varied.

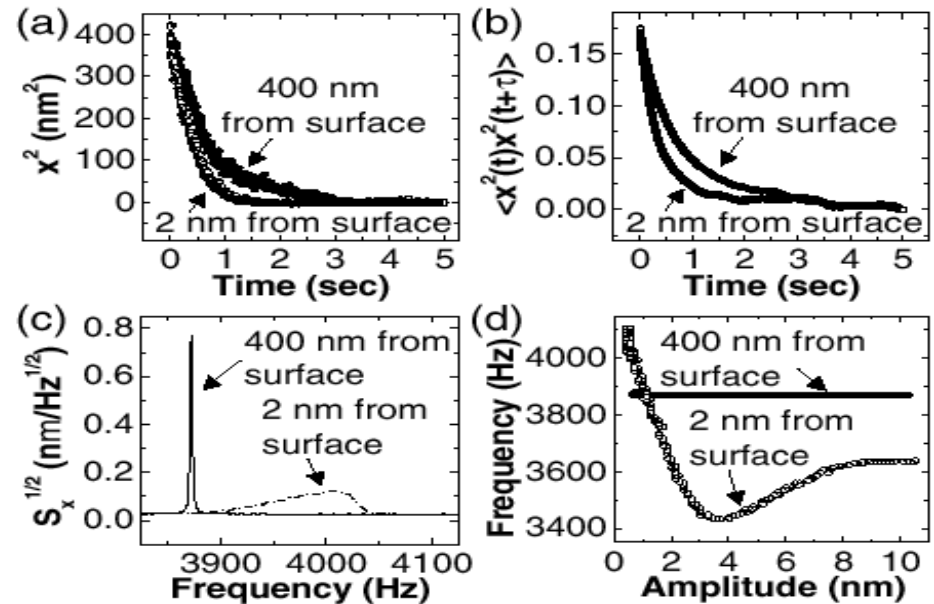


FIG. 2. Three methods for determining tip-sample friction. (a) Cantilever ringdown after excitation to an rms amplitude of 20 nm gives $Q = 17250$ for $d = 400$ nm and $Q = 8270$ for $d = 2$ nm. (b) Autocorrelation of the thermal energy $X^2(t)$ gives $Q = 20300$ for $d = 400$ nm and $Q = 8200$ for $d = 2$ nm (offset has been subtracted). (c) Spectral width of cantilever thermal vibrations gives $Q = 17000$ for $d = 400$ nm and $Q = 30$ for $d = 2$ nm assuming a harmonic potential. Notice, however, that the lineshape at 2 nm is not Lorentzian. (d) Cantilever frequency vs cantilever amplitude during free ringdown shows anharmonic behavior at $d = 2$ nm. All measurements were performed at 4.2 K.

Effect of electrostatic fields and Casimir force on cantilever vibrations

A.A. Chumak, P.W. Milonni, G.P. Berman (PRB 70, 085407 (2004))

We calculated the tip-sample attractive force due to the following mechanisms:
(a) Electrostatic force T_{voltage} between metallic tip and sample generated by bias voltage V .

Cylindrical tip

$$T_{\text{voltage}} = \frac{bR^{1/2}V^2}{2^{7/2}d^{3/2}} \quad (b - \text{width})$$

Spherical tip

$$T_{\text{voltage}} = \frac{RV^2}{4d}$$

Ellipsoidal tip

$$T_{\text{voltage}} = \frac{\sqrt{R_x R_y} V^2}{4d}$$

(b) Casimir force T_{casimir} due to fluctuations of electro-magnetic field in the tip-sample spacing (ω_p – plasma frequency).

Cylindrical tip

$$T_{\text{casimir}} = \frac{3bR^{1/2}\hbar\omega_p}{2^8 d^{5/2}}$$

Spherical tip

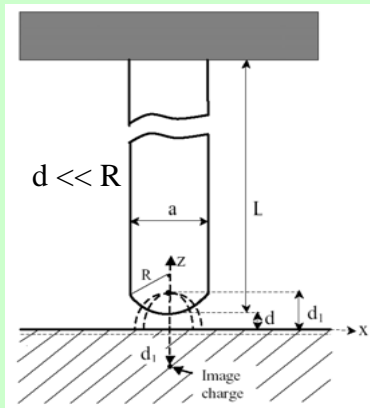
$$T_{\text{casimir}} = \frac{R\hbar\omega_p}{32\sqrt{2}d^2}$$

Ellipsoidal tip

$$T_{\text{casimir}} = \frac{\sqrt{R_x R_y} \hbar\omega_p}{32\sqrt{2}d^2}$$

(c) Attractive force due to spatial fluctuations of electric charge in dielectric sample.

Spherical tip: $T_{\text{defects}} = \frac{\pi c e^2 R}{2} \ln\left(1 + \frac{R}{8d}\right)$, c – concentration of defects.

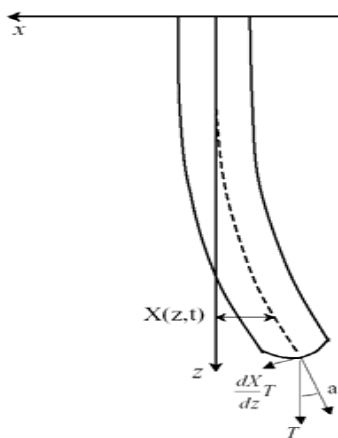


Cantilever-sample geometry

Results:

- Found explicit dependence of attractive force on voltage, distance, and tip geometry.
- Corrected previous results on Casimir force by Dorofeev *et al.* (PRB, 1999).
- Found that T_{defects} weakly depends on distance d , and may exceed the Casimir force.

Effect of attractive force T on fundamental frequency mode of cantilever

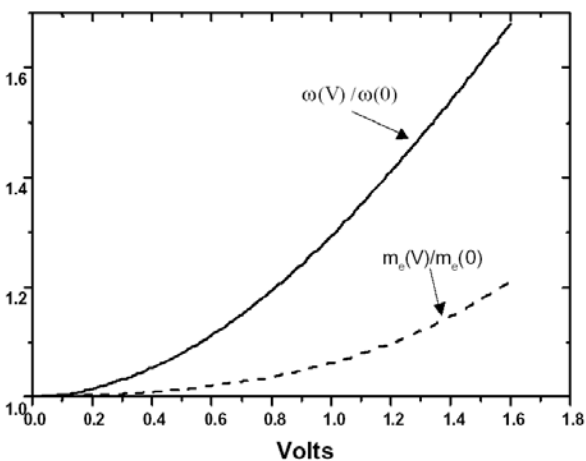


The transverse displacement $X(z,t)$ of the beam centerline satisfies the equation of elastic theory:

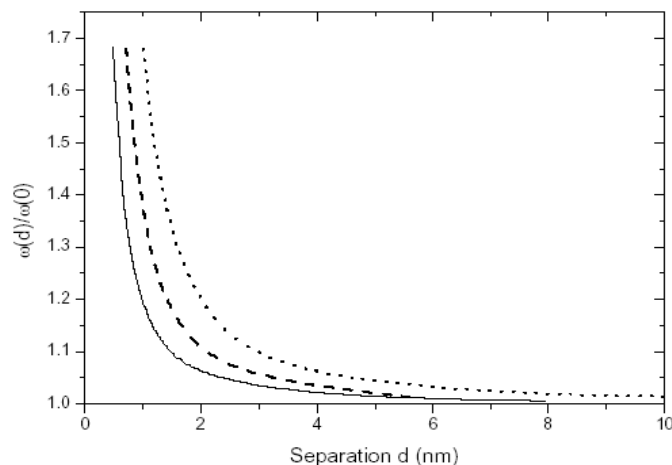
$$\rho S \frac{\partial^2}{\partial t^2} X(z,t) = -EI \frac{\partial^4}{\partial z^4} X(z,t) + T \frac{\partial^2}{\partial z^2} X(z,t),$$

and the boundary conditions at $z = L$: $-EIX''' + TX' = 0$, $X'' = 0$;

ρ , S , I , E — density, cross-section area, moment of inertia, Young's modulus of cantilever.



Dependences of cantilever frequency and effective mass on bias voltage for cylindrical geometry.

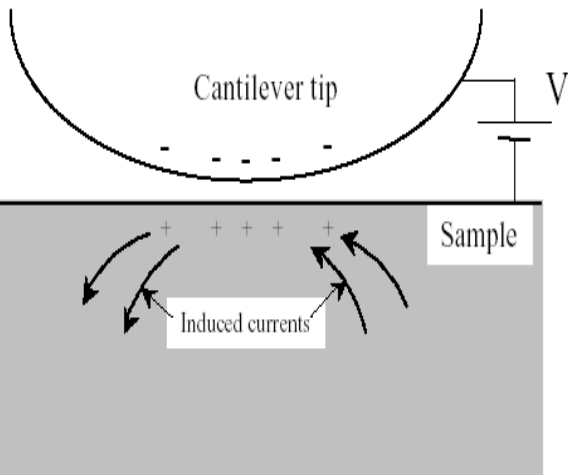


Frequency dependence on tip – sample separation in the case of dominating Casimir force. Quantitative agreement with experiment of Stipe *et al.* (PRB, 2001).

Results:

- Found dependences of cantilever fundamental frequency and effective mass on voltage and distance.
- Demonstrated that cantilever frequency shift in Stipe *et al.* (2001) can be explained by our results on dependence $\omega(d)$ for Casimir force.

Theory of Joule losses in cantilever-sample system with bias voltage



Arrows show the directions of recharging currents

Results:

- Found analytical expressions for Joule losses (W_J) and friction coefficient (Γ):

Cylindrical tip

$$W_j = \frac{b(VX_0\omega_c)^2}{2^7 \pi \mu d^2}$$

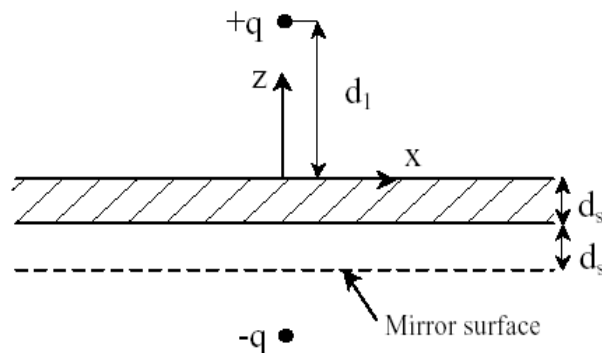
$$\Gamma = \frac{bV^2}{2^6 \pi \mu d^2}$$

Spherical tip

$$W_j = \frac{R^{1/2}(VX_0\omega_c)^2}{32\sqrt{2}\pi^3 \mu d^{3/2}} I$$

$$\Gamma = \frac{R^{1/2}V^2}{16\sqrt{2}\pi^3 \mu d^{3/2}} I$$

μ is a substrate conductivity, X_0 is the amplitude of oscillations, $I \approx 4.8$.



- In the case of finite thickness of the sample ($d_s < \sqrt{2Rd}$), Joule losses increase by factor $2d_l/d_s \gg 1$ due to increase of effective sample resistance.

CONCLUSIONS

Our analysis indicates that both electrostatic forces and Casimir forces can have a strong effect on the cantilever vibrations. The cantilever eigenfrequencies depend on the attractive force T , which is very sensitive to the tip geometry for small values of the tip-sample separations d . The effect of various forces on the cantilever eigenfrequencies appears to us to be an important consideration for the practical utilization of cantilever-based devices. It has served as a motivation in this research to study the tip-sample interaction in some detail. We have shown that, for small separations, T depends on the shape of the cantilever tip. The dependence of T on the separation, d , is different for cylindrical and spherical tips. Thus, in the case of the electrostatic force due to a bias voltage, the attractive force varies as $d^{-3/2}$ and d^{-1} for cylindrical and spherical tips, respectively. In the case of the Casimir force the dependence of T on d was found to be $d^{-5/2}$ and d^{-2} , respectively, for the cylindrical and spherical tips. We have shown that the Casimir force is possibly responsible for the frequency shift, which is of the same order as that obtained experimentally in B. C. Stipe, H. J. Mamin, T. D. Stowe, T. W. Kenny, and D. Rugar, Phys. Rev. Lett. 87, 096801 (2001), for a gold sample at small separation $d=2$ nm. The attractive force depends furthermore on the radii of curvature, and our analysis allows for the possibility that the radii may be different in different directions. In principle, any variations of tip geometry (for instance, due to adsorption of new molecules or blunting after contact with a sample) may be detected by measurements of the frequency shift, and our theory, which connects the variations of the attractive force with the frequency shift, could be helpful in estimating the character and scale of these variations. We have calculated the attractive force between a system of randomly distributed positive charges, embedded in a negatively charged background, and a metal tip. In the case of an electrically neutral system, only the attraction between each charge and its image contributes to the total force. This is a spatially fluctuating interaction because the overall force is linear (not quadratic) in the concentration of elementary charges. Our estimates, based on the assumption of an uncorrelated distribution of charge centers in the bulk of the substrate, indicate that the fluctuating force exceeds the Casimir force for parameters appropriate to the experiments reported in B. C. Stipe, *et al.*, which employed irradiated silica as a substrate. We have derived analytical expressions for Joule losses in a metal substrate. The Joule mechanism does not explain the cantilever damping measured in B. C. Stipe, *et al.*, but our analysis may nevertheless be useful for other systems with high-resistivity substrates.